Photon Aided and Inhibited Tunneling of Photons

Xuele Liu* and G.S. Agarwal
Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078, USA
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In the light of the interest in the transport of single photons in arrays of waveguides, fiber couplers, photonic crystals, etc., we consider the quantum mechanical process of the tunneling of photons through evanescently or otherwise coupled structures. We specifically examine the issue of tunneling between two structures when one structure already contains few photons. We demonstrate the possibility of both photon aided and inhibited tunneling of photons. The Bosonic nature of photons enhances the tunneling probability. We also show how the multiphoton tunneling probability can be either enhanced or inhibited due to the presence of photons. We find similar results for the higher order tunneling. Finally, we show that the presence of a squeezed field changes the nature of tunneling considerably.

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Quantum transport property of Fermions has been extensively studied in the last century [1, 2]. One interesting phenomenon of quantum transport of Fermions is the Coulomb blockade effect [3], which happens when electrons transport through a quantum dot. The Coulomb blockade effect inhibits tunneling by the presence of electrons in the quantum dot. Because of the Pauli exclusion principle for Fermions, each state of electrons my only contain one electron; incoming electron must occupy a higher energy level. The differences between energy levels in small quantum dots are very large (with the higher level far from the Fermi energy); therefore, the transport probability is very small and the tunneling is blocked.

An analog of the Coulomb blockade [4, 5] was demonstrated by Birnbaum *et. al* [6], showing that if an atom resonant with a strongly coupled single mode cavity could absorb one photon, then the absorption of second photon was inhibited. A similar experiment was reported in the context of superconducting qubits [7]. Another phenomenon is the dipole blockade [8] in Rydberg atoms where the excitation of a second atom to a Rydberg State is forbidden if one atom is already excited to a Rydberg state. The first excitation makes the second excitation non-resonant which leads to the blockade effect, and is similar to the Coulomb blockade in that it is based on the energy gap. The inhibited tunneling of photons that we discuss in this paper has a different origin. It not only depends on the energy gap but also on the Bosonic nature of photons. More generally in the context of transport of single photons (e.g. through arrays of waveguides or through lattices including photonic crystals) one may ask how the transport or tunneling characteristics depend on the presence of photons. Further one may ask how tunneling could depend on the quantum statistics of photons.

The simple and clear way to find the answer is to examine the photon tunneling between two or more coupled modes, with each mode referring to a different structure. This can be realized by coupled single-mode waveguide devices [9, 10] or by fiber couplers. The waveguides coupled by evanescent fields can be tailored by changing the distance between waveguides [9, 10]. The evanescent coupling is responsible for the tunneling of photons. Thus the transport of photons in this system is purely determined by how photons tunnel from one waveguide to the other.

In this letter we report a remarkable property of the tunneling of photons, i.e. the photon aided tunneling (PAT) and the photon inhibited tunneling (PIT), which can occur due to the presence of photons in the other waveguide. We find that even when the energy gap between the two waveguides is large, and the single photon tunneling is negligible, the PAT is significant and becomes about 1/e when the number of photons in the other waveguide is large. The PIT occurs when the energy gap is small. When the energy gap is zero, the tunneling rate without any photons in the other waveguide is 100%, the tunneling is totally inhibited when photons are present in the other waveguide. Both the PAT and PIT depend on the Bosonic nature of photons, i.e. each state may have more than one photon. It is not allowed for electrons due to the Fermi statistics. Our results are exact and go far beyond the perturbation theory.

We discuss the tunneling of photons between two coupled single-mode waveguides labeled A and B (Fig. 1), which can be realized by silica-on-silicon [9, 10]. Arrays of waveguides have been extensively studied with both classical and quantum light [9–15]. We can also use fiber couplers. We start with the simplest possibility that the waveguide A contains one photon and the waveguide B contains n photons; we calculate the probability $P_{(1,n)\to(0,n+1)}$ of the one photon from the waveguide A tunneling to the waveguide B. If $P_{(1,n)\to(0,n+1)} > P_{(1,0)\to(0,1)}$, then we conclude that the presence of n photons at waveguide B enhances tunneling. Experimentally, using an APD to detect the probability of having zero photons in waveguide A gives us a measurement of the tunneling of a single photon to the waveguide B.

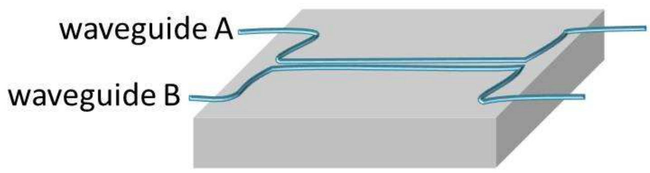

FIG. 1: Silica-on-silicon coupled waveguides [9].

Note that single photon technology is reasonably well developed and we now have an efficient source of single heralding photons [16]. Theoretically, the Hamiltonian for a system of two coupled waveguides is given by

$$\hat{H} = \Delta\left(a^{\dagger}a - b^{\dagger}b\right) + J\left(a^{\dagger}b + b^{\dagger}a\right) \tag{1}$$

here $a^{\dagger}$ ($b^{\dagger}$) is the operator that denotes creating a photon in waveguide A (waveguide B), and a (b) is the corresponding annihilation operator. Generally, the two waveguides A and B have different refractive indices $\varepsilon_0 \pm \Delta$. In the discussion of the time evolution of states, ε_0 only gives an overall phase $e^{-\mathrm{i}\varepsilon_0 t}$, which does not affect the transport probability. Thus, without loss of generality, we set $\varepsilon_0 = 0$. The coupling constant J is the tunneling energy of the photon to go from one waveguide to other.

If the system is initially in the state $\Psi(0) = |n, m\rangle$, i.e. there are n photons in the waveguide A, and m photons in the waveguide B, then at time t the system would be in state

$$\Psi(t) = \frac{\left(a^{\dagger}(-t)\right)^n \left(b^{\dagger}(-t)\right)^m}{\sqrt{n!m!}} |0,0\rangle . \tag{2}$$

The time dependent operators $a^{\dagger}(-t)$ and $b^{\dagger}(-t)$ can be obtained through the Heisenberg equation of motions. We have $\begin{bmatrix} a^{\dagger}(-t) \\ b^{\dagger}(-t) \end{bmatrix} = V(t)\begin{bmatrix} a^{\dagger}(0) \\ b^{\dagger}(0) \end{bmatrix}$, where $V(t) = e^{-\mathrm{i}Mt}$, with $M = \begin{bmatrix} \Delta & J \\ J & -\Delta \end{bmatrix}$. The $V(t)$ can be directly calculated from $e^{\mathrm{i}\sigma\cdot\mathbf{A}} = I_2 \cos A + \mathrm{i}(\sigma\cdot\mathbf{A})\frac{\sin A}{A}$. Here σ is the pauli matrix, A is the magnitude of vector $\mathbf{A}$, I_2 is the two dimensional identity matrix. We can then find

$$V(t) = \begin{bmatrix} \sqrt{1-P}e^{-\mathrm{i}\theta} & -\mathrm{i}\sqrt{P} \\ -\mathrm{i}\sqrt{P} & \sqrt{1-P}e^{\mathrm{i}\theta} \end{bmatrix}. \tag{3}$$

On defining $\gamma = \frac{\Delta}{J}$, $Q = \sqrt{1+\gamma^2}$ and $P_0 = \frac{1}{Q^2}$, the amplitude of the off-diagonal term of V is given by $\sqrt{P}$ with

$$P = P_0 \sin^2(QJt). \tag{4}$$

The amplitude of diagonal term is given by $\sqrt{1-P}$, and the corresponding phase θ can be calculated from probability $\sqrt{1-P}e^{-\mathrm{i}\theta} = \cos(QJt) - \mathrm{i}\frac{\gamma}{Q}\sin(QJt)$.

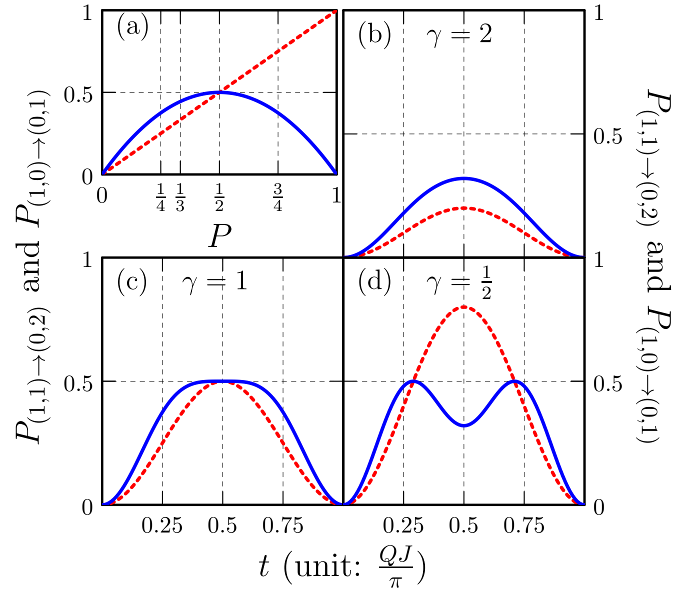

FIG. 2: The tunneling probabilities $P_{(1,0)\to(0,1)}$ (red dashed) and $P_{(1,1)\to(0,2)}$ (blue) as a function of (a). P; (b)-(d) scaled time $\frac{QJt}{\pi}$ for different values of $\gamma = \frac{\Delta}{J}$.

At time t, the probability that the system is in the state $|n', m'\rangle$ is given by $P_{(n,m)\to(n',m')} = |\langle n', m'|\Psi(t)\rangle|^2$. If the system is initially in the state $\Psi(0) = |1, n\rangle$, i.e. one photon in the waveguide A and n photons in the waveguide B, then the tunneling probability of the one photon from A to B is given by

$$P_{(1,n)\to(0,n+1)} = (n+1)(1-P)^n P. \tag{5}$$

The standard case of tunneling is a special case of Eq.(5) when there is no photon in waveguide B, i.e. $n = 0$. We may have $P_{(1,0)\to(0,1)} \equiv P$, (For brevity, we use P instead of $P_{(1,0)\to(0,1)}$ in the later discussion). Notice that this one-particle tunneling probability P is the same as in the Fermion case. In fact, we can see that, when Δ is large compared to J, then the energy gap between two modes is large (which means a high barrier at the junction), leading to large γ and a small tunneling probability. The small tunneling between the large gap is in fact the theoretical precondition of the Coulomb blockade of Fermions, it does not allow Fermions to tunnel to a state with much higher/lower than the Fermi energy.

In order to illustrate how the presence of photons within the other waveguide affects tunneling process, we first compare the tunneling probability $P_{(1,1)\to(0,2)}$ with the presence of one photon in the other waveguide with P. From Fig. 2.(a), we can see that when $P \leq 1/2$, $P_{(1,1)\to(0,2)} > P$, i.e. the presence of one photon in the waveguide B enhances the probability of tunneling, i.e. PAT occurs. It is easy to show from Eq.(5) that the maximum difference $(P_{(1,1)\to(0,2)} - P)_{\max} = 1/9$ is reached when $P = 1/3$. This means that the PAT occurs for any

range of values of the Δ and J. When the gap 2Δ is such that $\gamma = \frac{\Delta}{J} \geq 1$, we have $P_0 < 1/2$, $P < 1/2$, then the PAT always occurs. This is shown in Fig. 2.(b). Here $\gamma = 2$, $P_{(1,1)\to(0,2)} \geq P$ for any time t. When the energy gap is small $\gamma \leq 1$ thus $P_0 \geq 1/2$, we can still observe the PAT. The oscillation structure of P, Eq.(4) guarantees that there exist time regions such that $P \leq 1/2$ so that $P_{(1,1)\to(0,2)} \geq P$. (Fig. 2.(c) and Fig. 2.(d)).

From Fig. 2.(a), we also observe the appearance of the photon inhibited tunneling (PIT) when $P > 1/2$. Especially when $P = 1$ (100% tunneling probability without present any photon in waveguide B), we have exactly $P_{(1,1)\to(0,2)} = 0$, the photon tunneling is totally inhibited. The PIT can only be observed when the gap is small $\gamma < 1$ so that $P_0 > 1/2$, which allows $P > 1/2$ in some time region (Fig. 2.(d))).

The existence of PAT and PIT shows a competition mechanism introduced by the Bosonic nature of photons, as is seen from Eq.(5). Two factors are multiplied to P: the first one $(n+1) \geq 1$ is an aided term, which makes $P_{(1,n)\to(0,n+1)}$ possible to be bigger than P. It is from the Bosonic nature that $a^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$. The Bosonic nature shows that if a state contains more Bosons, then it is easier to add an extra Boson in it. The second term $(1-P)^n \leq 1$ is an inhibited term, which reflects the tendency that the n photons from waveguide B stay in the waveguide B. Notice that $(1-P)$ gives the probability that the one photon still wants to stay at its own site. The power n reflects the probability of the n-photon state to remain n-photon state. When n is larger, this term becomes smaller. The aided term $(n+1)$ is fixed when the photon number is fixed; while the inhibited term $(1-P)^n$ can decrease from 1 to 0 when P increases from 0 to 1. As a result: if P is small, the aided terms dominates and we observe the PAT; if P is large, then we observe the PIT.

In the light of this discussion, we examine $P_{(1,1)\to(0,2)}$. When $P = 0$, every photon must stay in its own state, the tunneling probability should be zero. When $P < 1/2$, the tunneling probability is small, every photon still wants to stay in its own state, the inhibited term $(1-P)^n$ is large. At this time the aided term $(n+1)$ is important, leading to $P_{(1,1)\to(0,2)} > P$. When $P > 1/2$, the inhibited term suppresses the positive effect of the aided term $(n+1)$.

We now discuss the general case of the tunneling probability $P_{(1,n)\to(0,n+1)}$ of one photon tunneling in the presence of n photons. From Fig.3.(a), we can see that when number of photons is increased, the region of PAT becomes smaller and occurs at the smaller values of P, however the maximum value does not decrease very much. In fact, from Eq.(5), we get $(P_{(1,n)\to(0,n+1)})_{\max} = (1 - \frac{1}{n+1})^n$ when $P = 1/(n+1)$. When the number of photons in waveguide B is very large $n \to \infty$, we can get $(P_{(1,n)\to(0,n+1)})_{\max} \to 1/\mathrm{e} \simeq 0.37$. Therefore we get very significant PAT especially when Δ is large which leads to

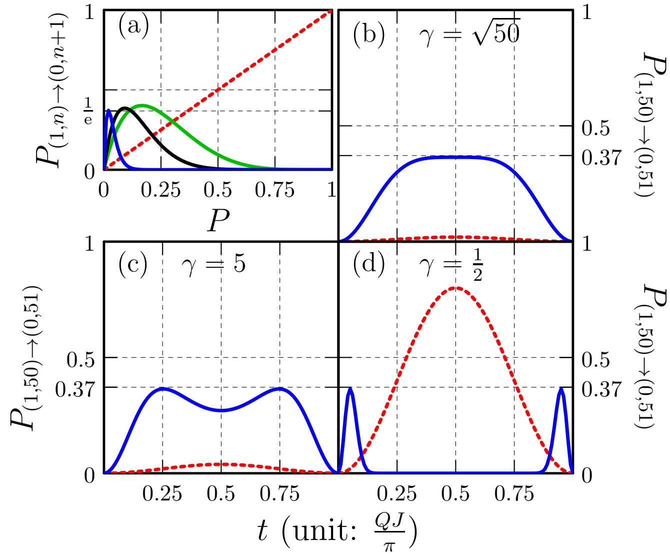

FIG. 3: (a). The tunneling probability $P_{(1,n)\to(0,n+1)}$ as a function of P for $n = 0$ (red dashed); $n = 5$ (green); $n = 10$ (black); $n = 50$ (blue). (b)-(d): The time dependence of the tunneling probability $P_{(1,50)\to(0,51)}$ (blue) as compared to that of $P_{(1,0)\to(0,1)}$ (red dashed) for different values of γ.

very small values of P. We compare the tunneling probability $P_{(1,n)\to(0,n+1)}$ with P for different γ. In Fig.3.(b), when $\gamma = \sqrt{n} = \sqrt{50}$ so that $P_0 = 1/51$, we can see that the tunneling without any presence of photons in the waveguide B is very small, while $P_{(1,50)\to(0,51)}$ has a plateau at 0.37 for a large time range; In Fig.3.(c), the aided tunneling is still significant for $\gamma = 5$. In Fig.3.(d), when the gap is small, the aiding tunneling acts as the pulse in the vicinity of time period T. This remarkable tunneling for large n has important application. This means that even for a large gap Δ, we can always find the finite photon tunneling probability $(P_{(1,n)\to(0,n+1)})_{\max}$ near $\frac{1}{\mathrm{e}}$ by choosing $n \geq \gamma^2 = \frac{\Delta^2}{J^2}$, whereas as for large Δ, $P_{(1,0)\to(0,1)}$ is negligible.

The above PAT can be generalized to multi-photon tunneling. In Fig.4, we compare $P_{(n_2,n)\to(0,n+n_2)}$ with $P_{(n_2,0)\to(0,n_2)} = P^{n_2}$. Obviously, $P_{(n_2,0)\to(0,n_2)}$ decreases when n_2 increases since $P < 1$. This is intuitive in that it is harder for more photons to tunnel to another waveguide. We can see that when $n_2 = 10$, the probability $P_{(n_2,0)\to(0,n_2)}$ is close to zero for most values of P. However, with photons in the waveguide B, the tunneling is significant. We can always find a finite maximum of $P_{(n_2,n)\to(0,n+n_2)}$. If $n \gg n_2$, this tunneling probability is even greater than the one-photon tunneling $P_{(1,0)\to(0,1)}$ as shown in Fig.4.

Using Eq. (2), we calculate the result for $P_{(n_2,n)\to(0,n+n_2)}$ to be:

$$P_{(n_2,n)\to(0,n+n_2)} = \begin{pmatrix} n+n_2 \\ n_2 \end{pmatrix} (1-P)^n\, P^{n_2}. \quad (6)$$

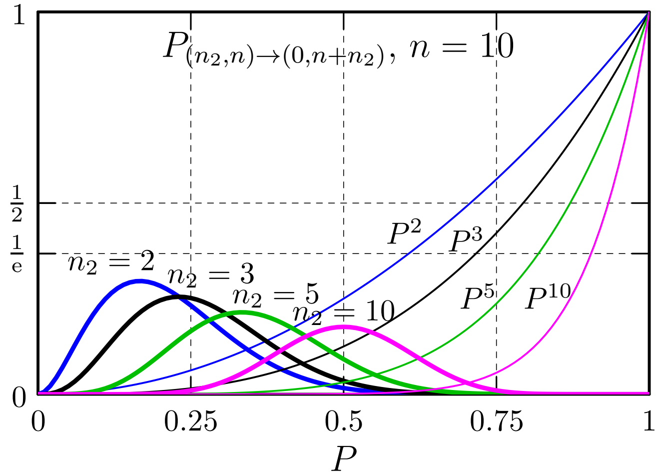

FIG. 4: The multiphoton tunneling probability $P_{(n_2,n)\to(0,n+n_2)}$ (thick lines) in presence of 10 photons in the waveguide B, as a function of P for $n_2 = 2, 3, 5, 10$. For each line, n_2 is marked at it's typical peak. The multiphoton tunneling probabilities without presence of photons in the waveguide B, $P_{(n_2,0)\to(0,n_2)} = P^{n_2}$ are given by thin lines.

where $\begin{pmatrix} n \\ m \end{pmatrix} = \frac{n!}{(n-m)!m!}$ is the binomial coefficient. From this equation, we can clearly see that the inhibited term is still given by $(1-P)^n$, while the aided term is changed to $\begin{pmatrix} n+n_2 \\ n_2 \end{pmatrix}$. This means the aided tunneling should be more important when n_2 increases. However, noticing that $P_{(n_2,0)\to(0,n_2)}$ decreases, the absolute value $P_{(n_2,n)\to(0,n+n_2)}$ may still decrease. This can be seen from Fig.4 which shows the decreasing peak of $P_{(n_2,n)\to(0,n+n_2)}$ with the increase of n_2. The maximum peak value is given by

$$\left(P_{(n_2,n)\to(0,n+n_2)}\right)_{\max} = \begin{pmatrix} n+n_2 \\ n_2 \end{pmatrix} \frac{n_2^{n_2} n^n}{(n+n_2)^{n+n_2}}. \quad (7)$$

which is reached when $P = \frac{n_2}{n+n_2}$. The limit when $n \to \infty$, is the maximum of $\frac{1}{n_2!}(\frac{n_2}{\mathrm{e}})^{n_2}$. It decreases when n_2 increases. However, even for $n_2 = 10$, we still have the finite tunneling about 12.5%, which is much greater then the corresponding tunneling probability $P_{(10,0)\to(0,10)} = \left(\frac{1}{20}\right)^{10}$ without the presence of any photons in the waveguide B.

In an experiment, it is much easier to prepare the field in a coherent state than a state with fixed photon number. The coherent state $|\beta\rangle = e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|n\rangle$ has the average photon number $\bar{n} = \langle\beta|\, b^\dagger b\,|\beta\rangle = |\beta|^2$. We may then discuss the possibility of the PAT and PIT with the field in coherent state in the waveguide B with a fixed average photon number $\bar{n}$. The probability of $P_{(n_2;\beta)\to(0;\beta,n_2)}$, n_2 photons tunneling from waveguide A to the waveguide B is given in terms of the confluent

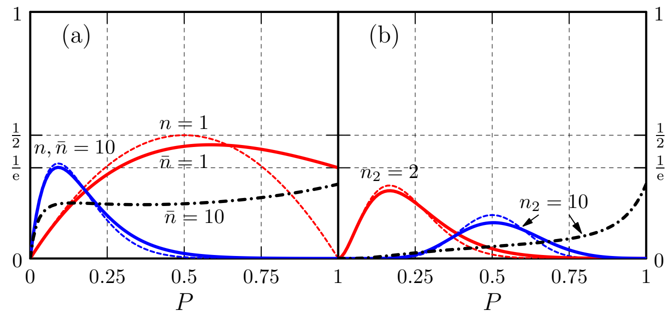

FIG. 5: Comparison of photon tunneling probabilities with field within waveguide B in a coherent state and in a Fock state. (a). The single photon tunneling probability $P_{(1;\beta)\to(0;\beta,1)}$ (solid lines) and $P_{(1;n)\to(0;n+1)}$ (dashed lines) as a function of P for $n, \bar{n} = 1, 10$; (b). The multiphoton tunneling probability $P_{(n_2;\beta)\to(0;\beta,n_2)}$ (solid lines) and $P_{(n_2,n)\to(0,n+n_2)}$ (dashed lines) in presence of (average) 10 photons in waveguide B, as a function of P, for $n_2 = 2, 10$. The black dashed dotted lines represent tunneling probability for a field in squeezed vacuum in waveguide B.

hypergeometric function ${}_1F_1\left[a, b; z\right]$:

$$P_{(n_2;\beta)\to(0;\beta,n_2)} = e^{-\bar{n}}\ {}_1F_1\left[1+n_2, 1, \bar{n}(1-P)\right] P^{n_2}. \quad (8)$$

In the special case of the one photon tunneling, the result is rather simple

$$P_{(1;\beta)\to(0;\beta,1)} = e^{-\bar{n}P}\left[1+\bar{n}(1-P)\right] P. \quad (9)$$

The formula for $P_{(1;\beta)\to(0;\beta,1)}$ and $P_{(n_2;\beta)\to(0;\beta,n_2)}$ are complicated, it's not easy to separate aided and inhibited terms. It's no doubt that $e^{-\bar{n}}$ and $e^{-\bar{n}P}$ are inhibited terms as they are always smaller than 1; however the second term contains both aided and inhibited parts. Even for one photon tunneling, the factor $\left[1+\bar{n}(1-P)\right]$ can be either greater or smaller than 1. However, from Fig.5, we observe that, the results for the coherent state case are very similar to the case of a Fock state in waveguide B, especially when $\bar{n} = n$ is large. We examine the maximum of $P_{(1;\beta)\to(0;\beta,1)}$, which is $\frac{\sqrt{\bar{n}^2+2\bar{n}+5}-2}{\bar{n}}\mathrm{e}^{\frac{1}{2}\left(\sqrt{\bar{n}^2+2\bar{n}+5}-\bar{n}-3\right)}$, and occurs for $P = \frac{\bar{n}^2+3\bar{n}-\bar{n}\sqrt{\bar{n}^2+2\bar{n}+5}}{2\bar{n}^2}$. For large $\bar{n}$, $\lim_{\bar{n}\to\infty}\left(P_{(1;\beta)\to(0;\beta,1)}\right)_{\max} = \frac{1}{\mathrm{e}}$, which is same as for the Fock state.

The tunneling probabilities are also sensitive to the photon statistics of photons in the waveguide B. To illustrate this, we consider the field in a squeezed state $|\xi\rangle = \frac{1}{\sqrt{\cosh r}}\sum_{n=0}^{\infty} e^{in\varphi}(\tanh r)^n \frac{\sqrt{(2n)!}}{n!2^n}|2n\rangle$ with $\overline{n} = \sinh^2 r = 10$. In this case, calculations show that

$$\begin{aligned} & P_{(n_2;\xi)\to(0;\xi,n_2)} && (10) \\ = \quad & \frac{P^{n_2}}{\sqrt{1+\overline{n}}}\ {}_2F_1\left[\frac{1+n_2}{2}, \frac{2+n_2}{2}, 1; (1-P)^2\frac{\overline{n}}{1+\overline{n}}\right], \end{aligned}$$

where ${}_2F_1\left[a, b, c; z\right]$ is the hypergeometric function. The dash-dotted black line in the Fig.5 shows the behavior of

(10) for $n_2 = 1$ and 10. The behavior is clearly different from the case of a coherent state: a long plateau occurs. We can conclude that the tunneling probability in the presence of a field in squeezed state $P_{(n_2;\xi)\to(0;\xi,n_2)}$ is mostly inhibited compared to the case when no field is present in waveguide B.

In conclusion we have shown how the tunneling of a single photon as well as multiphoton tunneling can be enhanced or inhibited by the presence of photons. We presented the physical reasons behind such an enhancement or inhibition. A crucial role is played by the Bosonic nature of photons. The waveguide structures or fiber couplers are known to be almost decoherence free, however if need be then the decoherence effects can be taken into account using the formulation of Ref. [17]. Although we explicitly considered the simplest case of a coupler the results can be extended to arrays of couplers. Tunneling is a universal effect in Physics; and therefore, results of this letter would be applicable to all situations in which Bosons are involved. Further the results of this paper should have a bearing on the quantum walk of a single photon in presence of other photons [18].

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* xuele@okstate.edu

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